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Transition from Asymmetric to Symmetric Tax Rules
— A Numerical Approach to Preferential Tax Regimes —

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Preferential tax regimes have been controversial among theorists. The relevant case is the competition between countries with certain differences. In the European context, countries such as Ireland and Germany may correspond. The present paper investigates, using numerical approach, the effects when countries adopting different tax rules (preferential and non-preferential) both adopts non-preferential tax regime (symmetric tax rule).

Keywords: preferential tax regimes; asymmetric countries; tax competition

I. Introduction

Tax competition for capital among countries has long been a major issue in international public policies and has a large literature of theoretical and empirical studies.[1)] In the traditional discussions, tax competition is considered to cause the "race to the bottom", where tax rates and government revenues are inefficiently decreased. On the other hand, the literature on preferential tax regimes is relatively new and is now growing. It is often observed that governments set lower tax rates on certain tax bases (or certain types of capital) than others to attract such tax bases into their jurisdictions. This is called tax discrimination or preferential tax regimes, which has been discussed among practitioners and theorists. In Ireland, for example, the corporate tax rate on certain industries such as finance and manufacturing was 10%, while that on other industries was 40%. Such policies, however, had been criticized by the European Commission and OECD as leading to harmful tax competition. In 2003, Ireland basically moved to the non-preferential regime, with a single tax rate of 12.5%.

Although it has been controversial among theorists whether preferential tax regimes are desirable or not,[2)] Keen's (2001) conclusion that tax preference can limit the scope of tax competition and increase tax revenues has been influential. While Keen (2001) assumed symmetric two countries, Bucovetsky and Haufler (2007) extended the model to asymmetric countries in population, assumed quadratic production functions, and showed that preferential tax regime generates larger

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tax revenues in both countries as in the symmetric model. Oshima (2009), on the other hand, showed that tax revenues can be larger under non-preferential regime using Cobb-Douglas production functions and assuming asymmetric productivities across countries. This shows that the result of Bucovetsky and Haufler (2007) which assumed symmetric quadratic production functions is not necessarily general but is a special case.

Theoretical studies of preferential tax regimes usually compare the cases where both countries adopt preferential tax regimes and where they both adopt non-preferential regimes (symmetric tax rules). This is because it is difficult to analytically investigate the equilibria where one country adopts preferential regime and the other adopts non-preferential regime, or the equilibrium under asymmetric tax rules.[3)] One can, however, calculate the equilibria by specifying the production functions and the values of parameters. In order to analyze the change in tax revenues of countries when a country like Ireland moved from preferential to non-preferential tax regime while other countries continued to adopt non-preferential regimes, we will have to study the case of asymmetric tax rules.

The present paper compares the equilibria under asymmetric tax rules and a symmetric rule (both countries adopt non-preferential regime). In section II we set up the model in general forms. Section III investigates the case using quadratic production functions. Section IV uses the Cobb-Douglas production functions and see the difference from the last section. Section V concludes.

II. The model

Suppose that the world consists of two countries, Country 1 and 2, and assume two types of mobile capital whose total amounts are fixed. The share of Country i in the total population is s^i, where $s^1 + s^2 = 1$. Each country has two representative firms (or industries), f and g. They employ labor (which is equal to population), as well as capital k and h, respectively. Labor is immobile across countries and industries. Country i has the same share of workers, s^i, in each sector. As usual in the analyses of asymmetric countries, let k^i denote the per-capita amount of capital k and h^i that of capital h employed in Country i. Therefore, the market clearing conditions of two types of capital imply:

$$s^1k^1 + s^2k^2 = \bar{k}, \qquad s^1h^1 + s^2h^2 = \bar{h} \tag{1}$$

where $\bar{k}$ and $\bar{h}$ are average amounts of per-capita capital k and h, respectively.

We assume here as in Keen (2001) that the location of each capital is unaffected by the tax on the other. The per-capita production functions are expressed as follows:

$$f^i = f(k^i), \qquad g^i = g(h^i).$$

Governments set tax rates t^i and T^i on the capital k^i and h^i, respectively. Therefore, from the profit maximization we have,

$$f_k^i = r + t^i, \qquad g_h^i = R + T^i$$

where subscripts denote partial derivatives, and r and R are the net-of-tax returns from the capital k and h, respectively. Because the net-of-tax returns are equalized across countries we have,

$$f_k^1 - t^1 = f_k^2 - t^2 \tag{2}$$

$$g_h^1 - T^1 = g_h^2 - T^2. \tag{3}$$

Substituting (1) into (2) and (3) and differentiating yields,

$$\frac{dk^i}{dt^i} = \frac{s^j}{s^j f_{kk}^i + s^i f_{kk}^j} \tag{4}$$

$$\frac{dh^i}{dT^i} = \frac{s^j}{s^j g_{hh}^i + s^i g_{hh}^j}. \tag{5}$$

Under preferential tax regimes certain tax base is discriminated ($t^i \neq T^i$) and governments solve the following revenue-maximization problem:

$$\max_{t^i, T^i} Rev^i = t^i k^i + T^i h^i.$$

Using (4) and (5) we have the following conditions:

$$k^i + \frac{s^j}{s^j f_{kk}^i + s^i f_{kk}^j} t^i = 0 \tag{6}$$

$$h^i + \frac{s^j}{s^j g_{hh}^i + s^i g_{hh}^j} T^i = 0. \tag{7}$$

Under non-preferential regimes each government levies a uniform tax rate τ^i on both tax bases and solve the following problem:

$$\max_{\tau^i} Rev^i = \tau^i (k^i + h^i)$$

which yields the condition as below:

$$k^i + h^i + (\frac{s^j}{s^j f^i_{kk} + s^i f^j_{kk}} + \frac{s^j}{s^j g^i_{hh} + s^i g^j_{hh}})\tau^i = 0. \tag{8}$$

Under asymmetric tax rules Country 1 adopts (6) and (7), while Country 2 follows (8).

III. Quadratic production functions

In this section we use quadratic production functions as in Bucovetsky and Haufler (2007), determine the values of parameters and calculate the equilibria when (i) population differs, (ii) productivity differs, and (iii) population and productivity differ across countries. The case (i) corresponds to the analysis by Bucovetsky and Haufler (2007), although in this section we compare the equilibria PN (preferential, non-preferential) and NN (non-preferential, non-preferential) as mentioned below.

1. Difference in population

We assume the following production functions,

$$f^i = 0.2k^i - 0.004(k^i)^2, \quad g^i = 0.2h^i - 0.002(h^i)^2, \quad i = 1, 2$$

and the allocation of population as $s^1 = 0.2$ and $s^2 = 0.8$, that is, we have small (Country 1) and large (Country 2) countries. Capital supplies are given as $\bar{k} = \bar{h} = 5$. We calculate two equilibria, where Country 1 adopts preferential tax regime and Country 2 adopts non-preferential tax regime (PN), and where both countries adopt non-preferential regime (NN), and see how tax rates and revenues change. The results are shown in Table 1.

Table 1: Difference in Population

	k^1	h^1	t^1	T^1	Rev^1
PN	7.50	12.50	0.08	0.06	1.34
NN	8.33	11.67	0.07	0.07	1.33

	k^2	h^2	t^2	T^2	Rev^2
PN	4.38	3.13	0.10	0.10	0.75
NN	4.17	3.33	0.10	0.10	0.75

Smaller country levies lower tax rates and attracts larger amounts of capital, as in Wilson (1991) and Bucovetsky (1991). Country 1's tax revenue decreases by its transition to non-preferential tax regime, while that of Country 2 is unchanged (which will be discussed later).

2. Difference in productivity

Next we consider the case where the productivity of industry f differs across countries and that of Country 1 is higher:

$$f^1 = 0.25k^1 - 0.002(k^1)^2, \quad f^2 = 0.2k^2 - 0.002(k^2)^2$$

$$g^i = 0.2h^i - 0.002(h^i)^2, \quad i = 1, 2$$

and the allocation of population is $s^1 = 0.5$, $s^2 = 0.5$, that is, two countries are of equal size. The equilibria are shown in Table 2. As in the last subsection, Country 1's tax revenue decreases by moving to non-preferential regime.

Table 2: Difference in productivity

	k^1	h^1	t^1	T^1	Rev^1
PN	7.60	4.48	0.06	0.04	0.62
NN	9.17	2.92	0.05	0.05	0.58

	k^2	h^2	t^2	T^2	Rev^2
PN	2.40	5.52	0.03	0.03	0.25
NN	0.83	7.08	0.03	0.03	0.25

3. Differences in population and productivity

Finally suppose population and productivity are different across countries. Production functions are the same as in the last subsection:

$$f^1 = 0.25k^1 - 0.002(k^1)^2, \quad f^2 = 0.2k^2 - 0.002(k^2)^2$$

$$g^i = 0.2h^i - 0.002(h^i)^2, \quad i = 1, 2$$

while the allocation of population is $s^1 = 0.2$ and $s^2 = 0.8$. Equilibria are shown in Table 3. Again, Country 1's tax revenue decreases by its transition to non-preferential regime.

Table 3: Difference in population and productivity

	k^1	h^1	t^1	T^1	Rev^1
PN	14.17	9.17	0.07	0.05	1.42
NN	16.67	6.67	0.06	0.06	1.36

	k^2	h^2	t^2	T^2	Rev^2
PN	2.71	3.96	0.07	0.07	0.44
NN	2.08	4.58	0.07	0.07	0.44

IV. Cobb-Douglas production functions

Bucovetsky and Haufler (2007) used quadratic production functions so that they can solve the problems analytically. In this section we use the Cobb-Douglas production functions which are more usual in economic analyses and see how the results are changed.

1. Difference in population

Assume the production functions as follows:

$$f^i = 1.5(k^i)^{0.3}, \quad g^i = (h^i)^{0.3}, \quad i = 1, 2$$

and the allocation of population as $s^1 = 0.2$ and $s^2 = 0.8$. Equilibria are shown in Table 4.

Table 4: Difference in population

	k^1	h^1	t^1	T^1	Rev^1
PN	8.03	11.43	0.13	0.11	2.32
NN	8.55	10.82	0.12	0.12	2.24

	k^2	h^2	t^2	T^2	Rev^2
PN	4.24	3.39	0.19	0.19	1.42
NN	4.11	3.54	0.18	0.18	1.40

By Country 1's transition to non-preferential tax regime, tax revenues decrease not just in Country 1 but also in Country 2. According to Keen (2001) preferential tax regimes limit the scope of fierce tax competition. Therefore, when production technologies are symmetric across countries, it seems natural that tax revenues of both countries decrease by a transition from equilibrium PN to NN and the tax competition being fiercer. The effect that Country 2's revenue also decreases is eliminated when we use quadratic production technologies because, as shown in (2), (3) and (6) – (8), the equilibria rest on first- and second-order differentiations of production functions which become linear and constants, respectively.

2. Difference in productivity

Next we consider the case where productivity differs across countries:

$$f^1 = 1.5(k^1)^{0.3}, \quad f^2 = (k^2)^{0.3}$$

$$g^i = (h^i)^{0.3}, \quad i = 1, 2$$

and $s^1 = 0.5$, $s^2 = 0.5$. The equilibria are shown in Table 5. One can see that tax revenues of

Table 5: Difference in productivity

	k^1	h^1	t^1	T^1	Rev^1
PN	5.35	5.13	0.18	0.14	1.68
NN	6.00	4.47	0.16	0.16	1.70

	k^2	h^2	t^2	T^2	Rev^2
PN	4.65	4.87	0.14	0.14	1.36
NN	4.00	5.53	0.15	0.15	1.41

both countries increase, instead of decrease, by Country 1's transition to non-preferential regime. We assumed here that f^1 is more productive than f^2 while g^1 and g^2 are symmetric. If we further change this so that g^2 is more productive than g^1 (say, replacing g^2 with $1.2(h^2)^{0.3}$) the increases of revenues become even larger. On the other hand, if you instead increase the productivity of f^2 the increases of tax revenues become smaller, and eventually negative because two countries approach symmetry, which is just as shown in Keen (2001). To see why tax revenues increase, note that the tax rates of Country 2 increase. This shows that the preferential tax regime in Country 1 no longer limits the scope of tax competition; Country 1's transition to non-preferential regime eases the competition when the productivities of some industry with Cobb-Douglas technologies are quite different across countries.

3. Differences in population and productivity

Suppose population and productivity are different across countries. Production functions are as follows:

$$f^1 = 1.5(k^1)^{0.3}, \quad f^2 = (k^2)^{0.3}$$

$$g^i = (h^i)^{0.3}, \quad i = 1, 2$$

while the allocation of population is $s^1 = 0.2$ and $s^2 = 0.8$. Equilibria are shown in Table 6. As in the last subsection, tax revenues of both countries increase by Country 1's transition to

Table 6: Difference in population and productivity

	k^1	h^1	t^1	T^1	Rev^1
PN	10.46	10.01	0.12	0.10	2.25
NN	11.63	8.89	0.11	0.11	2.27

	k^2	h^2	t^2	T^2	Rev^2
PN	3.63	3.75	0.16	0.16	1.16
NN	3.34	4.03	0.16	0.16	1.17

non-preferential regime. Therefore one can say that it is the difference in production technology that matters when considering the effects of a transition from PN to NN, as well as when both countries change the rule, as shown in Oshima (2009).

4. Games of two countries

Let us consider the games of two countries in strategic forms. Payoff matrices of the games of Tables 4 – 6 are expressed as Figures 1 – 3. In Figure 1 where two countries are asymmetric in population, the strategy pair (Preferential, Preferential), or PP, is the only (pure strategy) Nash equilibrium, which is consistent with Bucovetsky and Haufler (2007).

		Country 2	
		Preferential	Non-preferential
Country 1	P	$\underline{1.44}$ $\underline{2.28}$	1.42 $\underline{2.32}$
	N	$\underline{1.42}$ 2.23	1.40 2.24

Figure 1: Game of Table 4 (population)

		Country 2	
		Preferential	Non-preferential
Country 1	P	$\underline{1.37}$ $\underline{1.69}$	1.36 1.68
	N	1.39 1.68	$\underline{1.41}$ $\underline{1.70}$

Figure 2: Game of Table 5 (productivity)

In Figure 2 where two countries are asymmetric in productivity, we have two Nash equilibria although (Non-preferential, Non-preferential), or NN, is more desirable for both countries. This case corresponds to the result of Oshima (2009).

		Country 2	
		Preferential	Non-preferential
Country 1	P	$\underline{1.159}$ $\underline{2.272}$	1.157 2.254
	N	1.166 2.257	$\underline{1.169}$ $\underline{2.268}$

Figure 3: Game of Table 6 (population & productivity)

In Figure 3 where two countries differ in population and productivity, we again have two Nash equilibria. In this case, however, PP is more desirable for Country 1 while NN is preferable for Country 2. Therefore PN may occur temporarily. Whether it ends up with PP or NN will be determined by politics.

V. Conclusion

The present paper assumed two countries under asymmetric tax rules where Country 1 adopts preferential tax regime and Country 2 adopts non-preferential regime (PN). Then we calculated the results before and after Country 1 moves to non-preferential regime (NN). When two countries are asymmetric in population, Country 1's migration to non-preferential regime reduces the tax revenue of at least Country 1. When we assume Cobb-Douglas production functions and that productivities differ across countries, however, tax revenues may increase in both countries.

The results above are summarized in Table 7, which shows the asymmetries across countries and the effects of transitions from PN to NN. The relation between two countries such as Ireland and Germany may be close to any of [1], [3], [4], or [6] in the table. According to our results both countries may benefit from Ireland's transition to non-preferential regime. It requires, however, that the technologies of both countries be quite different.

Table 7: Asymmetries and the effects Country 1 moving to non-preferential regime

	population	production technology	both
quadratic functions	[1] decrease in Country 1's revenue	[2] same as on the left	[3] same as on the left
Cobb-Douglas functions	[4] both countries' revenue decrease	[5] both countries' revenue increase	[6] same as on the left

This suggests that previous studies on preferential tax regimes which assumed identical productivities across countries and compared PP and NN may not be appropriate to consider the case where one country moves from preferential to non-preferential tax regimes while others continue to adopt non-preferential regimes. It is not clear to which case ([1] – [6]) a specific relation of countries (e.g. Ireland and Germany) correspond or is similar. Hence, for that purpose, further researches such as empirical studies will be required.

Notes

1) See Wilson (1999), Zodrow (2003) and Wilson (2006) for surveys on tax competition.

2) See Oshima (2010) and Gaigné and Wooton (2011) for brief surveys on preferential tax regimes.

3) Gaigné and Wooton (2011) specify the production functions and analyzed the game in which tax rules are endogenously determined, and showed that countries adopt uniform taxes (non-preferential tax regimes) when trade costs are high.

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