流通科学大学論集—経済・情報・政策編—第 23 巻第 2 号，75-83（2015）

Mixed Oligopoly, Managers' Incentive, and Optimal Share of Government Ownership

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In earlier studies on mixed oligopoly where a public firm and private firms compete in a market, the government should have full ownership of the public firm if the number of private firms is endogenous (free entry). This does not hold, however, if the production costs of the firms depend on managers' incentives for cost reductions.

Keywords: mixed oligopoly; government ownership; privatization; free entry; production cost

I. Introduction

Mixed oligopoly is a market where a small number of firms, public and private, compete with each other. While governments around the world have promoted privatization of public firms, many public firms survived and compete with private firms in oligopolistic markets such as transportation, electricity, and finance. Therefore the importance of studying mixed oligopoly is increasing.

The studies of mixed oligopoly has a large literature. In most studies the public firm was assumed to maximize the social welfare which is the sum of consumer surplus and producers' profits, and the private firms to maximize profits, and the market was a quantity-setting oligopoly. At the early stage of the literature much attention was paid to the order of moves by public and private firms (whether the public firm is a Stackelberg leader or a follower, or the firms move simultaneously).[1)] It was assumed that the number of firms was fixed, i.e., there is no entry by new firms, with a few exceptions such as Ware (1986) and Sertel (1988). Main results were as follows. The government can increase the social welfare by appropriately controlling the public firm owned by the government. While the price of the good should equal the marginal cost in the first best, it is not optimal for some reason such as a budget constraint of the public firm.

In their seminal paper De Fraja and Delbono (1989) showed that, in the simultaneous-move mixed oligopoly with a fixed number of firms, the welfare can be higher when the public firm

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（2014 年 9 月 2 日受理）

maximizes its profit than when it aims to maximize welfare. This paradoxical result suggests that the public firm should be privatized in some cases. In addition, they showed that there exists a number, m, such that the welfare is higher when the public firm is privatized if and only if the number of private firms exceeds m. The results above can be explained as follows. Under privatization of the public firm the price is higher and the amount of goods produced is smaller than when the public firm maximizes welfare. Therefore the consumer surplus is smaller. On the other hand, the public firm produces more and its marginal cost is higher than private firms, and hence the production is inefficient. Therefore the total profit of producers is larger under privatization. The former effect (smaller consumer surplus) dominates the latter (larger profit of firms) if the number of private firms is small. The former becomes less important, however, as the number of private firms increase, and the latter dominates eventually. De Fraja and Delbono (1989) assumed that all the firms, public and private, have the same cost function. That is, the privatization of the public firm does not improve its efficiency. A remarkable contribution by them was to show that the privatization of the public firm can improve the social welfare even if it does not improve the efficiency of the public firm.

While De Fraja and Delbono (1989) considered the choice between full nationalization and full privatization, there are firms that are jointly owned by government and private firms, especially in industries such as transportation and electricity where privatization has been under way. Those firms are often relevant to mixed oligopoly. They have two targets, maximization of social welfare and maximization of their own profits, and are assumed to maximize the weighted average of these targets. Matsumura (1998) investigated such cases called "partial privatization" in mixed duopoly where a partially privatized public firm and a private firm operate in a market. He showed that under plausible conditions neither full nationalization nor full privatization is optimal, but partial privatization is desirable.

In the studies above it was assumed that the number of private firms was given exogenously. That is, there is no entry by new firms. This assumption was reasonable when public firms (including those partially privatized) were heavily regulated by the government. As the deregulation proceeds around the world, however, researchers increasingly needed to consider the effects of entry by new firms. Matsumura and Kanda (2005) extended the model above to cope with this issue. Private firms continue to enter the market until their profits equal zero. As a result, they showed that the public firm should produce goods so that the price equals its marginal cost,

and in order to do that the government should fully own the public firm. That is, partial or full privatization is not desirable.

In the models mentioned above they assumed that the productivities of public and private firms are identical or given exogenously and that privatization does not improve it. In the present paper we assume that the production costs of the firms depend on the incentives of the managers, and the public firm of which depends on the share of government ownership. While the results of empirical studies on this issue are divided, more recent studies find higher productivity as the share of private ownership increases.[2)] The rest of the paper is organized as follows. In section II we consider a model, based on Matsumura and Kanda (2005), in which the government share of ownership and production cost of the public firm are determined endogenously. In section III we specify all the functions and have numerical examples under different values of a parameter. Section IV concludes.

II. The Model

We consider a model that consists of three stages, in which one public firm and multiple public firms operate. The firms are indexed with $i = 0, 1, 2, ...n$ where $i = 0$ shows the public firm and others are private firms. At the first stage the government determines the ownership ratio of the public firm, and hence the influence of the government on the public firm. At the second stage managers (public and private) determine the levels of their human capital investments, which decreases the production costs while increasing their private benefits. At the third stage the firms play a Cournot-Nash game.

1. The first stage

At the first stage, the government determines the ownership ratio of the public firm, $s \in [0, 1]$. Then the influence of the government on the public firm, $\alpha \in [0, 1]$, is determined as the function of s. We assume,

$$\alpha(0) = 0, \quad \alpha(1) = 1, \quad \partial\alpha(s)/\partial s > 0.$$

2. The second stage

At the second stage, managers of the public and private firms determine the levels of their human capital investments, e_i. This investment decreases the production cost as explained later, and the

manager of a private firm can obtain a private benefit $b(e_i)$ as, for example, a bonus payment, where $b(0) = 0$, $b'(e_i) > 0$, and $b''(e_i) < 0$. Therefore the private manager solves the following maximization problem:

$$\max\ b(e_i) - e_i, \quad i = 1, 2, ..., n. \tag{1}$$

On the other hand, in the public firm, let us assume that the benefit to the manager is affected by α that shows the government influence, and the manager's maximization problem is as follows:

$$\max\ (1 - \alpha)b(e_0) - e_0. \tag{2}$$

Therefore we have the first order conditions as below:

$$(1 - \alpha)b'(e_0) = 1 \tag{3}$$

$$b'(e_i) = 1, \quad i = 1, 2, ...n. \tag{4}$$

That is, if $\alpha = 0$ (i.e., full privatization) the investment by the manager of firm 0, e_0, is equal to those by the managers of private firms $e_i\,(i = 1, 2, ..., n)$. On the other hand, if $\alpha = 1$ the public manager has no incentive to invest and we have $e_0 = 0$. If $0 < \alpha < 1$ we have $e_0 < e_i\,(i = 1, 2, ..., n)$.

3. The third stage

At the third stage, the firms play Cornout-Nash to produce the homogeneous good. The inverse demand function, $p = p(q)$, is twice-differentiable where $p > 0$ and $q \geq 0$, and satisfies $p'(q) < 0$. The cost function, denoted $c_i = c(x_i, \gamma_i)$, is common to all the firms, where x_i is the amount of good produced and $\gamma_i = \gamma(e_i)$ is the factor to reduce the cost and is a function of human capital investment that satisfies $\gamma_i' > 0$, $\gamma_i'' < 0$, and $\partial c_i/\partial \gamma_i < 0$. Hence, if $e_0 < e_i\,(i = 1, 2, ..., n)$ we have $c_0 > c_i$.

Social welfare, W, is the sum of consumer and producer surpluses and is expressed as below:

$$W = \int_0^X p(q)dq - \sum_{i=0}^{n} c_i$$

where,

$$X = \sum_{i=0}^{n} x_i. \tag{5}$$

Payoffs of public and private firms, U_0 and U_i, are as follows:

$$U_0 = \alpha W + (1-\alpha)[p(X)x_0 - c_0] \tag{6}$$

$$U_i = p(X)x_i - c_i. \tag{7}$$

Solving the maximization problems yields the first-order conditions as below:

$$(1-\alpha)p'x_0 + p - c_0' = 0 \tag{8}$$

$$p'x_i + p - c_i' = 0. \tag{9}$$

Private firms are homogeneous and we focus on symmetric equilibrium. Therefore at the equilibrium we assume,

$$x_1 = x_2 = ... = x_n. \tag{10}$$

Substituting (10) into (4), (5) and (9) we have,

$$b'(e_1) = 1 \tag{11}$$

$$X = x_0 + nx_1 \tag{12}$$

$$p'x_1 + p - c_1' = 0. \tag{13}$$

Under free entry the number of private firms is determined so that their profits equal zero. Therefore we have;

$$p(X)x_1 - c_1 = 0. \tag{14}$$

The equilibrium is obtained from (3), (8), (11) – (14).

III. Numerical Examples

In this section we specify the managers' benefit function, b_i, cost function, c_i, and cost-reducing factor, γ_i, as follows;

$$b_i = e_i^{1/2}, \quad c_i = 1 + x_i^2/\gamma_i, \quad \gamma_i = 1 + e_i^{1/2}/\phi, \quad \forall i, \tag{15}$$

where ϕ is a parameter, and the inverse demand function as,

$$p = 10 - X. \tag{16}$$

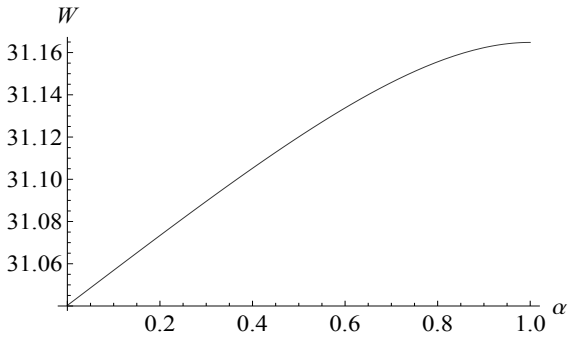

Figure 1: $\phi = 1000$

Then the social welfare W can be expressed as a function of the influence of government, α. As parameter ϕ increases, the second term of γ_i in (15) converges to zero and the cost function to $c_i = 1 + x_i^2$.

Figure 1 shows the welfare curve when ϕ is very large, that is, the effect of managers' incentives do not matter. Then the optimal value of α is unity, and hence full ownership of the public firm by the government is optimal as in Matsumura and Kanda (2005).

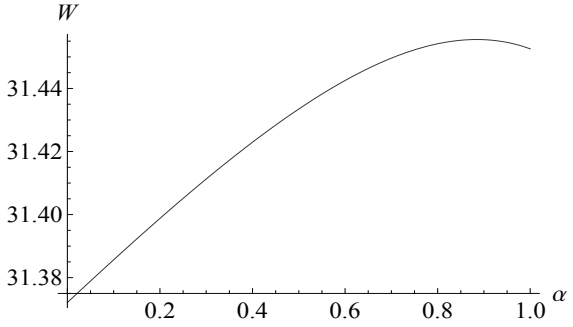

Figure 2: $\phi = 10$

Figure 2 shows the welfare curve when $\phi = 10$. As one can see, the optimal value of α is less than unity, and hence government's full ownership of the public firm is not optimal.

Figures 3 and 4 show the welfare curves when $\phi = 3$ and $\phi = 1$, respectively. One can see from Figure 3 that if the value of ϕ is intermediate it is optimal for the government to own about half of the public firm. In Figure 4, the optimal value of α is about 0.03 and it is zero when $\phi = 0.8$. That is, the larger the cost-reducing effect of human capital investment is, the more desirable the privatization is. If the effect is large enough, the advantage of public firm shown in earlier studies is all canceled out and full privatization is desirable.

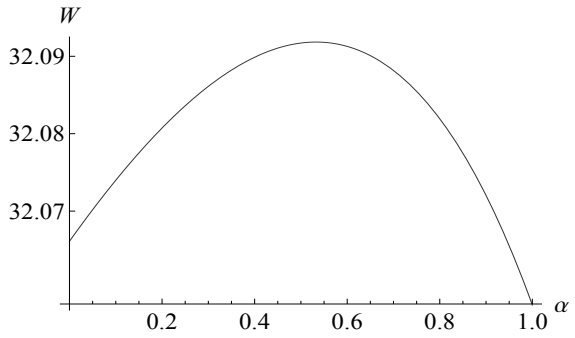

Figure 3: $\phi = 3$

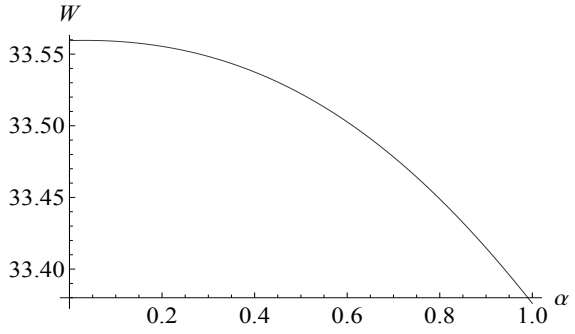

Figure 4: $\phi = 1$

One may say that the results above are not consistent with empirical studies. For example, Akai and Shinohara (2002) regressed the bankruptcy rate of quasi public enterprises in Japan on local government share of ownership and other variables. As a result, the bankruptcy rate was depicted as an inverted U-shaped curve as the government share increases. I do not see, however, that this result is inconsistent with the result of the current paper. First, the empirical study above uses average values of prefectures while the current study focuses on an individual public firm. In addition, while empirical studies are positive analysis the current study is a normative one using a stylized model. The reality may be viewed as various public firms with various values of ϕ, but relatively few public firms with intermediate value of ϕ.

IV. Conclusion

In the literature of mixed oligopoly De Fraja and Delbono (1989) showed that privatization of the public firm can increase social welfare. On the other hand, Matsumura and Kanda (2005) assumed free entry of new firms and showed that government should have full ownership of the

public firm. In the present paper we assumed that the ownership of the public firm by the private sector increases the manager's incentive to reduce production cost, and extended the model of Matsumura and Kanda (2005) so that the production cost and the ownership are determined endogenously. The result is as follows. As the parameter ϕ in the cost function becomes larger the effort by the manager is less effective. If ϕ is large enough the result is the same as in Matsumura and Kanda (2005): full ownership by the government is optimal. As ϕ decreases, the optimal share of government ownership becomes lower, that is, partial privatization is optimal. If ϕ is small enough full privatization is desirable.

While the result above may not be very surprising, we can see that the condition for full government ownership may not be met easily (ϕ has to be very large) given that the manager's effort can reduce production cost. The desirability of (partial) privatization will depend on the production technologies of the industries.

Another issue is that public firms in reality often do not conduct just one task but do multiple tasks. Some of them may be suitable for privatization while others may not. For example, power companies own power plants and grid systems, the former of which is better suited for privatization while the latter is not. Similarly, operating vehicles of railroad companies may be better suited for privatization than maintaining rails and signals. In those industries public firms can be separated and parts of them may be privatized. This issue is left for future research.

Notes

1) See De Fraja and Delbono (1990) for a survey.

2) Bös (1991), section 3.2, discusses this issue.

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